Can a Particle's Velocity Exceed the Speed of Light in Empty Space?

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Relative motion in space with multifractal time (fractional dimension of time close to integer $d_t = 1 + \varepsilon(\mathbf{r}, t)$, $|\varepsilon| \ll 1$) for "almost" inertial frames of reference (time is almost homogeneous and almost isotropic) is considered. Presence in such space of absolute frames of reference and violation of conservation laws (though, small because of the smallness of ε) due to the openness of all physical systems and inhomogeneiy of time are shown. The total energy of a body moving with $v = c$ is obtained to be finite and modified Lorentz transformations are formulated. The relation for the total energy (and the whole theory) reduce to the known formula of the special relativity in case of transition to the usual time with dimension equal to unity.

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I. INTRODUCTION

As well known, the special relativity theory (SR) is the theory of inertial systems and for such systems the answer to the question posed in the title of this paper is negative. But in the nature ideal inertial systems do not exist. It allows to raise the following problem: is it possible to develop a theory of systems close to inertial ("almost" inertial systems) that would include, as a special case, special relativity but, at the same time, would allow for a motion of particles with any velocity? Obviously, in order to invent such a theory it is necessary to refuse from the rigorous validity of any of the SR postulates: the homogenity of space and time, the invariance of speed of light and the Galilee invariance principle.

The present paper suggests an example of such a theory, based on the concepts of time and space with fractional dimensions (FD) developed in the theory of multifractal time and space [1]. We begin with the first of the mentioned SR principles. In an inhomogeneous space and time, if the inhomogenities are small enough, any motion will be close to that in homogeneous space ("almost" inertial), but the velocity of light can alter slightly, being thus "almost" constant. Mere assumption that the values of fractional dimensions of space and time are close to integer leads then to conclusion that the motions of particles with any velocities become possible. Other main assumptions made are the light velocity invariance and invariance with respect to modified Lorentz transformations. The results our theory gives for the velocities less than the velocity of light c almost coincide with SR, but it does not contain singularities at $v = c$. Stress, that this theory is not a generalization of SR theory, because any such generalization in the domain of SR validity (inertial systems) is absurd. Our theory describes relative movements only in the "almost" inertial systems, and thus does not contradict to SR.

II. MULTIFRACTAL TIME

Following [1], we will consider both time and space as the only material fields existing in the world and generating all other physical fields. Assume that every of them consists of a continuous, but not differentiable bounded set of small elements (elementary intervals, further treated as "points"). Consider the set of small time elements S_t. Let time be defined on multifractal subsets of such elements, defined on certain measure carrier $\mathcal{R}^n$. Each element of these subsets (or "points") is characterized by the local fractional (fractal) dimension (FD) $d_t(\mathbf{r}(t), t)$ and for different elements FD are different. In this case the classical mathematical calculus or fractional (say, Riemann - Liouville) calculus [2] can not be applied to describe a small changes of a continuous function of physical values $f(t)$, defined on time subsets S_t, because the fractional exponent depends on the coordinates and time. Therefore, we have to introduce integral functionals (both left-sided and right-sided) which are suitable to describe the dynamics of functions defined on multifractal sets (see [1]). Actually, this functionals are simple and natural generalization the Riemann-Liouville fractional derivatives and integrals:

$$D^d_{+,t} f(t) = \left(\frac{d}{dt}\right)^n \int_a^t \frac{f(t')dt'}{\Gamma(n - d(t'))(t - t')^{d(t')-n+1}} \quad (1)$$

$$D^d_{-,t} f(t) = (-1)^n \left(\frac{d}{dt}\right)^n \int_t^b \frac{f(t')dt'}{\Gamma(n - d(t'))(t' - t)^{d(t')-n+1}} \quad (2)$$

where $\Gamma(x)$ is Euler's gamma function, and a and b are some constants from $[0, \infty)$. In these definitions, as usually, $n = \{d\} + 1$, where $\{d\}$ is the integer part of d if $d \geq 0$ (i.e. $n - 1 \leq d < n$) and $n = 0$ for $d < 0$. If $d = const$, the generalized fractional derivatives (GFD)

(1)-(2) coincide with the Riemann - Liouville fractional derivatives ($d \geq 0$) or fractional integrals ($d < 0$). When $d = n+\varepsilon(t)$, $\varepsilon(t) \to 0$, GFD can be represented by means of integer derivatives and integrals. For $n = 1$, that is, $d = 1 + \varepsilon$, $|\varepsilon| << 1$ it is possible to obtain:

$$D^{1+\varepsilon}_{+,t} f(t) \approx \frac{\partial}{\partial t} f(t) + a \frac{\partial}{\partial t} \left[\varepsilon(r(t), t) f(t)\right] \tag{3}$$

where a is constant and defined by the choice of the rules of regularization of integrals (1)-(2) (for more detailed see [1]). The selection of the rule of regularization that gives a real additives for usual derivative in (3) yeilds $a = 0.5$ for $d < 1$ and $a = 1.077$ for $d > 1$ [1]. The functions under integral sign in (1)-(2) we consider as the generalized functions defined on the set of the finite functions [3]. The notions of GFD, similar to (1)-(2), can also be defined for the space variables $\mathbf{r}$.

The definitions of GFD (1)-(2) are formal until the connections between fractal dimensions of time $d_t(\mathbf{r}(t), t)$ and certain characteristics of physical fields (say, potentials $\Phi_i(\mathbf{r}(t), t)$, $i = 1, 2, ..$) or densities of Lagrangians L_i) are determined. Following [1], we define this connection by the relation

$$d_t(\mathbf{r}(t), t) = 1 + \sum_i \beta_i L_i(\Phi_i(\mathbf{r}(t), t)) \tag{4}$$

where L_i are densities of energy of physical fields, β_i are dimensional constants with physical dimension of $[L_i]^{-1}$ (it is worth to choose β'_i in the form $\beta'_i = a^{-1}\beta_i$ for the sake of independence from regularization constant). The definition of time as the system of subsets and definition (4) put the value of fractional (fractal) dimensionality $d_t(r(t), t)$ into accordance with every time instant t. The latter depends both on time t and coordinates $\mathbf{r}$. If $d_t = 1$ (absence of physical fields) the set of time has topological dimensionality equal to unity. The multifractal model of time allows, as will be shown below, to consider the divergence of energy of masses moving with speed of light in the SR theory, as the result of the requirement of rigorous validity, rather than approximate fulfillment, of the laws pointed out in the beginning of this paper in the presence of physical fields.

III. THE PRINCIPLE OF THE VELOCITY OF LIGHT INVARIANCE

Because of the inhomogeneity of time in our multifractal model, the speed of light, just as in the general relativity theory, depends on potentials of physical fields that define the fractal dimensionality of time $d_t(\mathbf{r}(t), t)$ (see (4)). If fractal dimensionality $d_t(\mathbf{r}(t), t)$ is close enough to unity ($d_t(r(t), t) = 1 + \varepsilon$, $|\varepsilon| << 1$), the difference of the speed of light in moving (with velocity v along the x axis) and fixed frame of reference will be small. In the systems that move with respect to each other with almost constant velocity (stationary velocities do not exist in the mathematical theory based on definitions of GFD (1) - (2)) the speed of light can not be taken as a fundamental constant. In the multifractal time theory the principle of the speed of light invariance can be considered only as approximate. But if ε is small, it allows to consider a nonlinear coordinates transformations from the fixed frame to the moving (replacing the transformations of Galilee in inhomogeneous time and space), as close to linear (weakly nonlinear) transformations and, thus, makes it possible to preserve the conservation laws, and all the invariants of the Minkowski space, as the approximate laws. Then the way of reasoning and argumentation accepted in SR theory (see for example, [4]) can also remain valid. Designating the coordinates in the moving and fixed frames of reference through x' and x, accordingly, we write down

$$\begin{aligned} x' &= \alpha(t, x)[x - v(x, t)t(x(t), t] \\ x &= \alpha'(t, x)[x' + v'(x'(t'), t'),\ t'(x'(t'), t') \end{aligned} \tag{5}$$

In (5) $\alpha \neq \alpha'$ and the velocities v' and v (as well as t and t') are not equal (it follows from the inhomogenity of multifractal time). Place clocks in origins of both the frames of reference and let the light signal be emitted in the moment, when the origins of the fixed and moving frames coincide in space and time at the instant $t' = t = 0$ and in points $x' = x = 0$. The propagation of light in moving and fixed frames of reference is then determined by equations

$$x' = c't' \;\; x = ct \tag{6}$$

These characterize the propagation of light in both of the frames of reference at every moment. Due to the time inhomogenity $c' \neq c$, but since $|\varepsilon << 1|$ the difference between velocities of light in the two frames of reference will be small. For this case we can neglect the distinction between α' and α and, for different frames of reference write the expressions for velocities of light, using (3) to define velocity (denote $f(t) = x$, $dx/dt = c_0$). Thus we obtain

$$c = D^{1+\varepsilon}_{+,t} x = c_0(1 - \varepsilon) - \frac{d\varepsilon}{dt} x \tag{7}$$

$$c' = D^{1+\varepsilon'}_{+,t} x' = c_0(1 - \varepsilon) + \frac{d\varepsilon}{dt} x' \tag{8}$$

$$c_1 = c_0(1 - \varepsilon) - \frac{d\varepsilon}{dt} x' \tag{9}$$

$$c'_1 = c_0(1 - \varepsilon) + \frac{d\varepsilon}{dt} x \tag{10}$$

The equalities (9) and (10) appear in our model of multifractal time as the result of the fact, that in this model all the frames of reference are absolute frames of reference (because of material character of the time field) and

the speed of light depends on the state of frames: if the frame of reference is a moving or a fixed one, if the object under consideration in this frame moves or not. This dependence disappears only when $\varepsilon = 0$. Before substitution the relations (5) in the equalities (7) - (10) (with $\alpha' \approx \alpha$) it is necessary to find out how $d\varepsilon/dt$ depends on α. Using for this purpose Eq.(4) we obtain:

$$\frac{d\varepsilon}{dt} = \frac{d\varepsilon}{d\mathbf{r}}\mathbf{v} \approx -\sum_i \beta_i(\mathbf{F}_i\mathbf{v} + \frac{\partial L_i}{\partial t}) \tag{11}$$

where $\mathbf{F}_i = dL_i/d\mathbf{r}$. Since the forces for moving frames of reference are proportional to α we get (for the case when there is no explicit dependence of L_i on time)

$$\frac{d\varepsilon}{dt} \approx -\sum_i \beta_i \mathbf{F}_{0i}\mathbf{v}\alpha \tag{12}$$

where F_{0i} are the corresponding forces at zero velocity. Multiplying (7) - (10) on the corresponding times t, t', t_1, t_1' yields the following expressions

$$c't' = c_0 t\left[1 + \frac{v\sum_i \beta_i F_{0i}}{c_0}\alpha^2 ct(1 - \frac{v}{c})\right] \tag{13}$$

$$ct = c_0 t'\left[1 + \frac{v\sum_i \beta_i F_{0i}}{c_0}\alpha^2 ct(1 + \frac{v}{c})\right] \tag{14}$$

$$c_1' t_1' = c_0 t_1\left[1 - \frac{v\sum_i \beta_i F_{0i}}{c_0}\alpha^2 ct(1 - \frac{v}{c})\right] \tag{15}$$

$$c_1 t_1 = c_0 t_1'\left[1 - \frac{v\sum_i \beta_i F_{0i}}{c_0}\alpha^2 ct(1 + \frac{v}{c})\right] \tag{16}$$

Since in our model the motion and frames of reference are absolute, the times t_1 and t_1' correspond to the cases, when the moving and fixed frames of reference exchange their roles - the moving one becomes fixed and vice versa. These times coincide only when $\varepsilon = 0$. The times in square brackets, as well as the velocities, are taken to equal, because the terms containing them are small as compared to unity. The principle of invariance of the velocity of light for transition between the moving and fixed frames of reference in multifractal time model is approximate (though quite natural, because the frames of reference are absolute frames of reference). Taking into account (5), the relations (13) - (16) take the form

$$c't' = c\alpha t(1 - \frac{v}{c}), \quad c_1' t_1' = c\alpha t_1(1 - \frac{v}{c}) \tag{17}$$

$$ct = c\alpha t'(1 + \frac{v}{c}), \quad c_1 t_1 = c\alpha t'(1 + \frac{v}{c}) \tag{18}$$

Once again we note, that the four equations for $c_1' t_1'$ and $c_1 t_1$, instead of the two equations in special relativity, appear as the consequence of the absolute character of the motion and frames of reference in the model of multifractal time. In the right-hand side of (17) - (18) the dependence of velocity of light on fractal dimensions of time is not taken into account (just as in the equations (13) - (16)). Actually, this dependence leads to pretty unwieldy expressions. But if we retain only the terms that depend on $\beta = \sqrt{|1 - v^2/c^2|}$ or a_0 and neglect unessential terms containing the products $\beta\alpha_0$, utilizing (13) - (16) after the multiplication of the four equalities (17) - (18), we receive the following equation for α (it satisfies to all four equations):

$$4a_0^4\beta^4\alpha^8 - 4a_0^2\alpha^4 + 1 = \beta^4\alpha^4 + 4a_0^4\beta^4\alpha^8 \tag{19}$$

where

$$\beta = \sqrt{\left|1 - \frac{v^2}{c^2}\right|} \tag{20}$$

$$a_0 = \sum_i \beta_i F_{0i}\frac{v}{c}ct \tag{21}$$

From (19) follows

$$\alpha_1 \equiv {\beta^*}^{-1} = \frac{1}{\sqrt[4]{\beta^4 + 4a_0^2}} \tag{22}$$

The solutions $\alpha_{2,3,4}$ are given by $\alpha_2 = -\alpha_1$, $\alpha_{3,4} = \pm i\alpha$. Applicability of above obtained results is restricted by requirement $|\varepsilon| \ll 1$

IV. LORENTZ TRANSFORMATIONS AND TRANSFORMATIONS OF LENGTH AND TIME IN MULTIFRACTAL TIME MODEL

The Lorentz transformations, as well as transformations of coordinate frames of reference, in the multifractal model of time are nonlinear due to the dependence of the fractional dimensions of time $d_t(\mathbf{r}, t)$ on coordinates and time. Since the nonlinear corrections to Lorentz transformation rules are very small for $\varepsilon \ll 1$, we shall take into account only the corrections that eliminate the singularity at the velocity $v = c$. It yields in the replacement of the factor β^{-1} in Lorentz transformations by the modified factor $\alpha = 1/\beta^*$ given by (22). The Lorentz transformation rules (for the motion along the x axis) take the form

$$x' = \frac{1}{\beta^*}(x - vt), \quad t' = \frac{1}{\beta^*}(t - x\frac{v}{c^2}) \tag{23}$$

In the equations (22) and (23) the velocities v and c weakly depend on x and t and their contribution to the singular terms is small. Hence, we can neglect this dependence. The transformations from fixed system to moving system are almost orthogonal (for $\varepsilon \ll 1$), and the

squares of almost four-dimensional vectors of Minkowski space vary under the coordinates transformations very slightly (i.e. they are almost invariant). Then it is possible to neglect the correction terms of order about $O(\varepsilon, \dot{\varepsilon})$, which, for not equal to infinity variables, are very small too. From (22) - (23) the possibility of arbitrary velocity motion of bodies with nonzero rest mass follows. With the corrections of order $O(\varepsilon, \dot{\varepsilon})$ in nonsingular terms being neglected, the momentum and energy of a body with a nonzero rest mass in the frame of reference moving along the x axis ($E_0 = m_0c^2$) equal to

$$p = \frac{1}{\beta^*} m_0 v = \frac{m_0 v}{\sqrt[4]{\beta^4 + 4a_0^2}}, \quad E = E_0 \sqrt{\frac{v^2 c^{-2}}{\sqrt{\beta^4 + 4a_0^2}} + 1} \tag{24}$$

The energy of such a body reaches its maximal value at $v = c$ and is equal then $E_{v=c} \approx E_0/\sqrt{2\alpha_0}$. When $v \to \infty$ the energy is finite an tends to $E_0\sqrt{2}$. For $v \leq c$ the total energy of a body is represented by the expression

$$E \cong \frac{E_0}{\sqrt[4]{\beta^4 + 4a_0^2}} = mc^2, \quad m = \frac{m_0}{\beta^*} \tag{25}$$

For $v \geq c$, total energy, defined by (24), is given by

$$E = mc^2, \quad \beta^2 = \frac{v^2}{c^2} - 1$$
$$m = \frac{m_0}{\beta^*} \sqrt{2v^2/c^2 + 4a_0^2} \tag{26}$$

If we are to take into account only the gravitational field of Earth (here, as in [5], gravitational field is a real field) and neglect the influence of all the other fields), the parameter $a_0(t)$ can be estimated to be $a_0 = r_0 r^{-3} x_E ct$, where r_0 is the gravitational radius of Earth, r is the distance from the Earth's surface to its center ($\varepsilon = 0.5\beta_g\Phi_g$, $\beta_g = 2c^{-2}$, $x_E \sim r$, $v = c$). For energy maximum we get $E_{max} \sim E_0 \cdot 10^3 t^{-0.5} sec^{0.5}$.

Shortening of lengths and time intervals in moving frames of reference in the model of multifractal time also have several peculiarities. Let l and t be the length and time interval in a fixed frame of reference. In a moving frame

$$l' = \beta^* l, \quad t' = \beta^* t \tag{27}$$

Thus, there exist the maximal shortening of length when the body's velocity equals to the speed of light. With the further increasing of velocity (if it is possible to fulfill some requirements for a motion in this region with constant velocity without radiating), the length of a body begins to grow and at infinitely large velocity is also infinite. The slowing-down of time, from the point of view of the observer in the fixed frame (maximal shortening equals to $t' = t\sqrt{2a_0}$) is replaced, with the further increase of velocity beyond the speed of light, by acceleration of time passing ($t \to 0$ when $v \to \infty$).

The rule for velocities transformation retains its form, but β is replaced by β^*

$$u_x = \frac{u'_x + v}{1 + \frac{u'_x v}{c^2}}, \; u_y = \frac{u'_y \beta^*}{1 + \frac{u'_y v}{c^2}}, \; u_z = \frac{u'_z \beta^*}{1 + \frac{u'_z v}{c^2}} \tag{28}$$

Since there is no law that prohibits velocities greater than that of light, the velocities in (28) can also exceed the speed of light. The electrodynamics of moving media in the model of multifractal time can be obtained, in most cases, by the substitution $\beta \to \beta^*$.

V. CONCLUSIONS

To conclude, the theory of relative motions in almost inertial systems based on the multifractal time theory [1] is invented. This theory describes open systems (for statistical theory of open systems see in [6]) and in this theory motion with any velocity is possible. The theory coincides with special relativity after transition to inertial systems (if we neglect the fractional dimensions of time) or almost coincides (the differences are negligible) for velocities $v < c$. Movement of bodies with velocities that exceed the speed of light is accompanied by a number of physical effect's which can be found experimentally (these effects will be considered in the separate paper in more detail).

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